Local (static) universe model

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Abstract. Stationarity of our Universe is a precondition for the existence of Life. The proposed paper reviews some early attempts to build a model of global stationary Universe. We suggest the mathematical model of the local stationary universe and discuss the properties of the Weierstrass function that gives its solution.

Key words. *Universe, model, local universe, Weierstrass function.*

1. Introduction

Theoretical cosmology originated as part of the general theory of relativity when Einstein proposed the first mathematical model of static Universe in 1916. It is fair to point out and recognize the fundamental contribution by Newton, as well as the first fundamental work of Jeans that was published under the title *The stability of a spherical nebula* in 1902.

Shortly after, Friedmann proved that the Einstein equations have nonstationary solutions, and besides, working on the basis of astronomical observation, Hubble discovered in 1929 the recession of the galaxies. Advances in the development of modern technologies, breakthroughs in particle physics and space exploration led to adoption of the so-called Big Bang model.

But there are islets of stationary local universes inside the well-established picture of global expanding Universe. The discovery of such universes coincided with the emergence of the revolutionary concepts of dark energy and dark matter.

At present, the main efforts of cosmologists are aimed at the development of the theory of global Universe, right up to the hypothetical mirror and parallel worlds, while the models of local universes (one of which we live in) are still unjustly overlooked. Our goal is to draw the attention of the contemporary cosmologists to our home stationary world. Such stationarity is essential to the existence of Life.

2. Early models of stationary Universe

The well-known Einstein model was proposed in his paper [1]. Before using of the general relativity equation, Einstein gave several prophetic pointers. Long before this was realized by other scientists [2], [3], [4], he clearly mentioned that: 1) it is enough to use Newtonian approximation in cosmology; 2) the matter is evenly distributed in large areas of space; 3) we can introduce the cosmological constant Λ in order to counterbalance the expansion and obtain a static Universe.

Basing on this, Einstein modified his general relativity equations in the style similar to the generalized Poisson equation and achieved the stationary model in the form

$$R_{ik} - \Lambda g_{ik} = -\varkappa \left(T_{ik} - \frac{g_{ik}}{2} \right).$$

This model meets all the criteria in both physical and mathematical aspects, but it turns out to be unstable, according to the result of Friedmann [5], [6]. Thus, it is more suitable for the case of the global expanding Universe, living up to all expectations.

Einstein's theoretical model lacks one more assumption to give stability to our local stationary universe. Bondi, Gold, Hoyle and Narlikar came very close to the point in their Steady State Theory. They note that the Einstein - Friedmann's model has something that is extremely important — it has what is called a singularity problem. The problem might be avoided by imposing a hypothetical C - field with negative energy density, and this was the intent behind Hoyle - Narlikar's (HN) model as a Steady State Theory [8].

The original idea of stationary global Universe with continuous creation of baryonic matter was suggested simultaneously in papers by Bondy - Gold [7] and Hoyle [8]. More detailed elaboration of these principles was given by Hoyle and Narlikar [9], [10], [11], [12]. The main point to be made here is the hypothesis of the existence of the C - field providing the creation of matter according to the model

$$\frac{d\varrho_m}{dt} = -\,3H\varrho_m + A, \quad \frac{d\varepsilon_C}{dt} = -\,C^2 A - 3H(\varepsilon_C + P_C).$$

Taking $A = 3H\varrho_m$ one can fulfil the stationary condition.

In 1963 Hoyle and Narlikar [13] assumed that matter is created predominantly in areas of high pressure (sic!). For more than two decades since 1948, much of the work has been devoted to verification, development and critical thinking around the HN - model.

Unfortunately, the complete set of available data does not confirm it. Zeldovich and Novikov [19], however, pay tribute to intellectual courage of the authors of this theory, since the discussion surrounding it was helpful and contributed to the overall progress of cosmology.

It should be added that the idea of the continuous creation of matter has not exausted its role because certain hopes can be placed presently on non-baryonic dark matter, the origin of which remains a great mystery.

3. Observation data

Starting from 1994, very interesting observation data were obtained by Karachentsev's team of astronomers [14], [15], [16]. According to these observations, about 40 local galaxies together with the halo of dark matter form a system with the center near the Milky Way and the Andromeda Galaxy. The secession of a part of the halo has been observed.

The gravitational pull of this local group neutralizes the antigravitational effect of dark energy. As a consequence, our local group appears to be quasistationary. Some 20 dwarf galaxies manage to overcome the attraction of our local group and form the so-called Hubble flow which is subordinated to the general law of galaxies recession.

It is significant to note that the total mass of dark matter in the local group greatly exeeds the total amount of its baryonic matter, and this gives us grounds for considering galaxies as test particles inside the field of dark energy; it is expected that the most part of dark matter is located in voids. It was also noted that the older galaxy you get, the greater is the density of dark matter associated with it. We do not know what it is made of, but, according to the current theoretical understanding, it produces some gravitational potential wells that capture baryonic matter.

The structure of Universe required for the birth of living matter is formed in this manner. Different nearby groups of galaxies with their halos constitute superclusters in a form of pancakes of approximately 30 Mpc. And finally, they shape chains or filaments of superclusters, each in number of 5 - 20.

A reference should be made to the works by Shaya et al. [18] that gave the full computer simulation of trajectories over the last 13 billions of years for a nearly complete set of galaxies in our local universe. Their attempt was based on data from the CosmicFlows - 2 survey.

4. Model of local universe

It was natural to ask for what reason the local universe must be stationary (or, more precisely, quasistationary, as it contains the nonstationary Hubble flow). Cut this flow off, and we obtain for a long period a stationary local universe within a 1.5 Mpc radius of it. To a first approximation, such universe is stabilizing with the help of the countervailing influences of gravitational and antigravitational forces (cf. Einstein's model). Dark baryonic matter provides some additional stability in accordance with the HN - model. Mindful of its lessons, one can only conjecture the continuous birth of invisible *non - baryonic* matter, and that happens where the density of matter is maximal, i. e. inside the galaxies.

A novel mathematical model of our local stationary universe is presented below. We came to it through a synthesis of Einstein's and Hoyle - Narlikar's theories. In particular, we assume the hypothesis of the exponential creation of dark matter in the depths of galaxies. It is invisible matter which is responsible for causing the halos.

ϱ - model. Let us consider the quasistationary group of galaxies immersed in the halo of dark matter and suppose that the mass of dark matter had grown exponentially in its evolution

$$\frac{dM}{dt} = \alpha M,$$

where α is the relative speed of growth.

Thus, our first equation

$$\frac{d\varrho}{dt} = \alpha\varrho - 3H\varrho$$

could be established along the lines of Friedmann's theory [19]. Here we take

$$\varrho(t) = \frac{M}{V}, \quad V = \frac{4\pi R^3}{3}.$$

Furthermore, employing Newton's Law of Gravitation with account of dark energy in the form of Λ - term

$$\frac{d^2R}{dt^2} = -\frac{\gamma M}{R^2} + \frac{c^2\Lambda R}{3},$$

we can derive the second equation

$$\frac{dH}{dt} = -H^2 - \frac{4\pi\gamma\varrho}{3} + \frac{c^2\Lambda}{3}.$$

Here, γ stands for the gravitational constant, c means the speed of light, Λ signifies the cosmological constant and H refers to the Hubble constant.

Bringing together both equations, we come to the nonlinear law

$$\frac{d^2\varrho}{dt^2} - (\alpha - 3H)\frac{d\varrho}{dt} + (3H^2 - c^2\Lambda)\varrho - 4\pi\gamma\varrho^2 = 0$$

that describes the dynamics of global Universe. Integration of baryonic matter mass will not change the general form of this equation. Stationarity occurs under the conditions

$$\alpha = 3H, \quad 3H^2 = c^2\Lambda,$$

that is to say, first order terms should disappear.

In this case the main equation

$$\frac{d^2\varrho}{dt^2} - 4\pi\gamma\varrho^2 = 0$$

looks very simple, while its solution describes the density of dark matter

$$\varrho(t) = \wp(t; g_2, g_3)$$

and is given by the famous Weierstrass elliptic function.

Particularly, we get the so-called *equiharmonic case* for the values of invariants $g_2 = 0$ and $g_3 = 1$. Then $\varrho(t)$ is almost constant or stationary for a long period of time. See [21] for the flattened plot of this function which stretches along the half-axis $0 < t < \infty$. One can try to describe the *inflaton* with the aid of such function.

R**-model.** Let us use the cartesian coordinates in order to visualize the process. For this purpose, the equation should be integrated against the factor dR/dt:

$$\frac{1}{2}\left(\frac{dR}{dt}\right)^2 + \frac{\gamma M}{R} - \frac{c^2\Lambda R^2}{6} = C.$$

The value of constant

$$C = \frac{H_0^2 R_0^2}{2} - \frac{4\pi\gamma R_0^2 \varrho_0}{3} - \frac{c^2\Lambda R_0^2}{6}$$

can be identified in terms of the values H_0, R_0 and ϱ_0 at the initial moment t_0. Hence, we get

$$\left(\frac{dR}{dt}\right)^2 = \frac{8\pi\gamma\varrho_0 R_0^3}{3R} + \frac{\Lambda c^2(R^2 - R_0^2)}{12} - \frac{4\pi\gamma R_0^2(\varrho_0 - \varrho_C)}{3},$$

where

$$\varrho_C = \frac{3H_0^2}{4\pi\gamma}.$$

The solution of the last equation may be expressed via the energy integral.

5. Discussion

We built a new mathematical model that requires the experimental verification and the development of a theoretical basis. It helps to obtain a qualitative insight into the whole picture: *the radius of local universe depends on* ϱ_0/ϱ *and* R^2/R_0^2 *and does not change till the density* $\varrho(t)$ *changes.* To account for the data then available, two drastic conjectures had to be made: we have accepted the hypothesis about the origin of the halo and the concept of the exponential growth of invisible matter.

The elliptic function of Weierstrass remains the focal point of the project. This special function is difficult to define in terms of elementary ones. The process of exponential growth itself leads us to believe that some biological mechanisms lay behind this phenomenon. In this regard, the Weber-Fechner law (the logarithmic relation between the actual stimulus and the noticeable change) is of interest. Either way, it is not an accident, but rather a Law of Nature.

It has long been noted that a merciful fate had started following our Universe from its very origin. As a result of phase transitions of vacuum, it went straight to the minimum of potential energy; elementary particles and fields were born correctly and promptly. Hoyle [23] firmly believed that the carbon - oxigen synthesis is all premeditated by a Superintellect. Novikov [24] wrote about this matter that the whole Universe looks like it was perfectly fine - tuned for Life to exist. A similar reasoning applied to the fundamental constants also led to the concept of the Intelligent Design. On the other hand, the disagreement between the observed small value of the cosmological constant

$$\Lambda = \Lambda_1 - \Lambda_2$$

and the large value of vacuum energy suggested by the quantum field theory is known as the vacuum catastrophe. We must presume the existence of two vacua with energy densities Λ_1 and Λ_2 which are almost equal in value but opposite in sign.

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